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PRC
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(Codes et Séries formelles)

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Traitements Algébriques et Informatiques des Séries Formelles Non Commutatives

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1. Systèmes dynamiques et calcul formel

the local minimal realization of non commutative power series. We immediately realize that it is a tough task. Thus we restrict our study to non commutative polynomials, using the *Lyndon basis* of the *free Lie-algebra* and *Lyndon words* in order to construct local coordinates.

We present some definitions and properties of non commutative power series, Lyndon words and dynamical systems. In the end of this paper, we present a complete execution of our software which is described in the computer algebraic system Macsyma.

2. Etudes algébriques

Gérard CAUCHON (Reims) :

Séries de Malcev sur le groupe libre et questions de rationalité

Consider a field k, a free group G with basis $\mathcal{B}$ and denote by Ω the free submonoid generated by $\mathcal{B}$. The theory of formal languages and formal power series consists in the study of the algebra $k\langle\langle\Omega\rangle\rangle$ of power series with support in Ω and coefficients in k and, in particular, of the recognizable formal power series which are, by the Kleene-Schützenberger theorem (1961), the rational ones, that is to say the elements of $k\,rat\langle\langle\Omega\rangle\rangle$, the smallest subalgebra of $k\langle\langle\Omega\rangle\rangle$ containing the associative free algebra $k\langle\Omega\rangle$ of the series with finite support and having the property :

If $s \in k\,rat\langle\langle\Omega\rangle\rangle$ has zero valuation, then $s^{-1} \in k\,rat\langle\langle\Omega\rangle\rangle$.

It has been proved by Birkhoff in 1946 that the free group G can be (fully linearly) ordered and the theory of Malcev-Neumann series consists in the study of the algebra $k_M((G))$ of power series with well ordered support in G and coefficients in k. By a theorem due to B.H. Neumann (1948), this algebra turns out to be a division ring. As it contains the group algebra $k[G]$, one may define the division ring $k(G)$ of rational power series with support in G and coefficients in k, as the smallest division ring in $k_M((G))$ which contains $k[G]$. As pointed by B.H. Neumann, the division ring $k(G)$ has a rather complicated and mysterious structure.

The purpose of this paper is to prove that the two notions of rationality defined above turn out to coincide. In fact, we have :

$$k\langle\langle\Omega\rangle\rangle \cap k(G) = k\,rat\langle\langle\Omega\rangle\rangle$$

The proof of that result uses the construction of sub-division rings of $k_M((G))$, much bigger than $k(G)$, but easier to control. It also uses a combinatorial study of the canonical mapping $\varphi : G \longrightarrow G/D_2$ where D_2 is the second derivated group of the free group G.

Gérard DUCHAMP – Daniel KROB (Rouen) :

Free partially commutative structures

In this paper, we study several algebraic structures, called "free partially commutative structures", that are defined by a presentation of the form : $< A\,;\ ab = ba \ \text{ for } (a,b) \in \theta >$ where θ is a given subset of $A \times A$. We show that these relators are the only one that can be both interpreted as monoid and Lie relators. We also study the free partially commutative monoids, Lie algebra and groups : in all of these cases, we show how to obtain decomposition results into the corresponding free structures. In particular, we give a generalization of Lazard's elimination theorem for the free partially commutative Lie algebra that permits us to construct combinatorial bases of the underlying free $\mathbb{Z}$-module of this Lie algebra.

3. Combinatoire

Marie-Pierre DELEST – Jean-Marc FEDOU (Bordeaux) :

Attribute grammars are useful for combinatorics

The purpose of this paper is to show the use of attribute grammars in solving some combinatorics problems. For example, we give the generating function for the shape of skew Ferrer diagrams according to the number of columns. This result is new and proves that skew Ferrer diagrams are related to new basic Bessel functions.